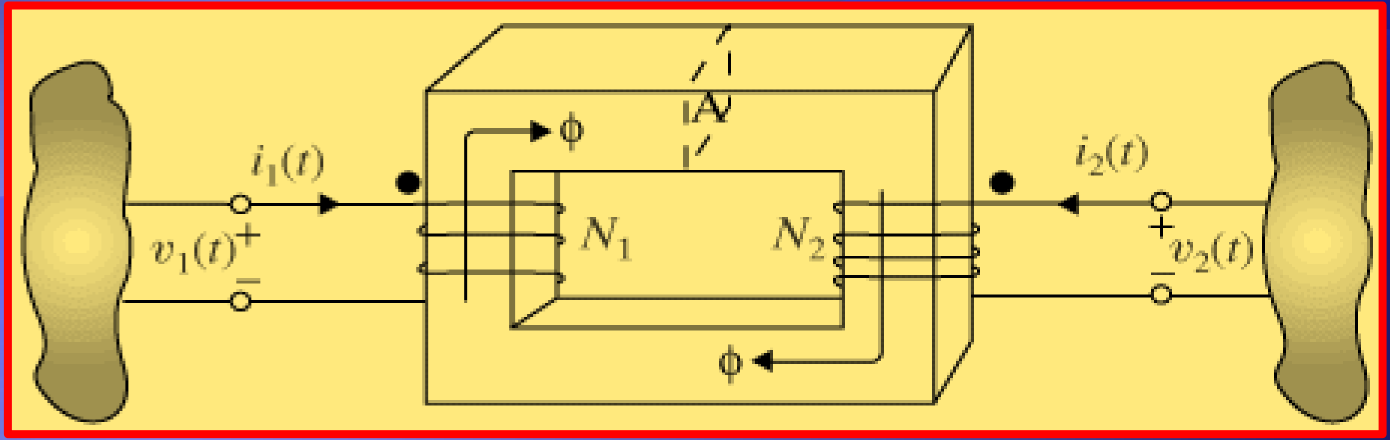
IDEAL TRANSFORMER



If we consider the particular situation where the two coupled coils are wound around a single closed magnetic core, with the **same core flux, ϕ, linking all the turns of both coils**, we could write:

$$v_1(t) = N_1 \frac{d\phi}{dt} \text{ and} : v_2(t) = N_2 \frac{d\phi}{dt}$$

Which means that:

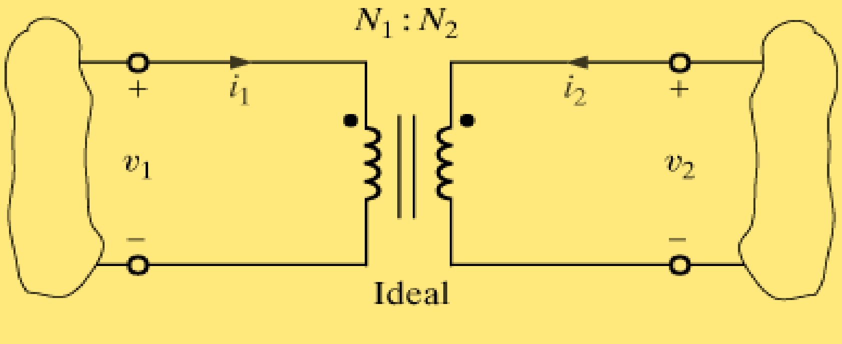
$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

On the other hand, if we consider that the **wire resistance of the coils is negligible**, we deduce that the transformer under consideration is **lossless**:

$$v_1(t) i_1(t) + v_2(t) i_2(t) = 0$$

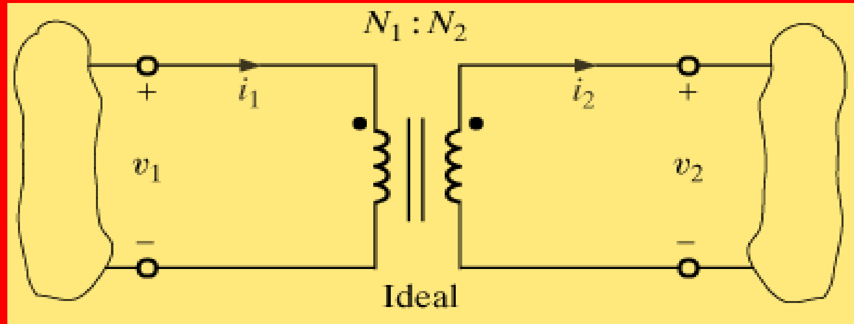
$$\Rightarrow \frac{i_1}{i_2} = -\frac{N_2}{N_1}$$

These equations are applicable when **both currents are flowing into the dots and both voltages are positive at the dots**, as shown in the following Figure:



(a)

However, the normal power flow through such coil combination takes place **from an input current in the primary to an output current in the secondary**, as shown in the following Figure. In this case, the corresponding voltage and current equations could be written as:



(b)

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

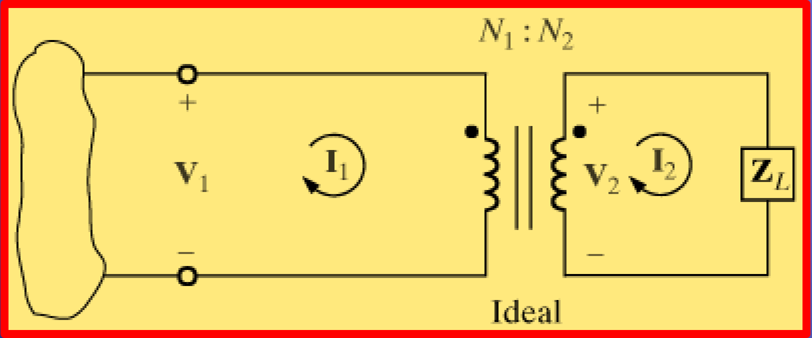
$$\text{and} : \frac{i_1}{i_2} = \frac{N_2}{N_1}$$

In both cases, such coil combination is called **Ideal Transformer**. It is modeled using the word “Ideal” under the two coils, **two vertical lines** representing the common core, and **two numbers N_1 and N_2** representing the **turns ratio** of the transformer.

We note that although the voltage and current levels change through an ideal transformer, **the power levels do not!**

Note that **practical transformers** do not use dots as such. Instead, they use **markings** specified in the US by the National Electrical Manufacturers Association (**NEMA**). These markings are conceptually equivalent to the dots.

Now, if we consider the following phasor-domain circuit:



Since **both voltage phasors are positive at the dots**, we could write:

$$\boxed{\frac{\mathbf{V_1}}{\mathbf{V_2}} = \frac{N_1}{N_2}}$$

And since **one current phasor enters the dot while the other leaves the dot**:

$$\boxed{\frac{\mathbf{I_1}}{\mathbf{I_2}} = \frac{N_2}{N_1}}$$

In other words:

$$\boxed{\mathbf{V_1} = \frac{N_1}{N_2}\mathbf{V}_2 \text{ and } \mathbf{I_1} = \frac{N_2}{N_1}\mathbf{I_2}}$$

Therefore, the input impedance is:

$$\boxed{Z_1 = \frac{\mathbf{V_1}}{\mathbf{I_1}} = \left(\frac{N_1}{N_2}\right)^2 \frac{\mathbf{V_2}}{\mathbf{I_2}} = \left(\frac{N_1}{N_2}\right)^2 Z_L}$$

Which means that the load impedance, ***Z_L* is reflected** into the primary side **by the square of the turns ratio**.

If we designate the turns ratio as n:

$$\boxed{n = \frac{N_2}{N_1}}$$

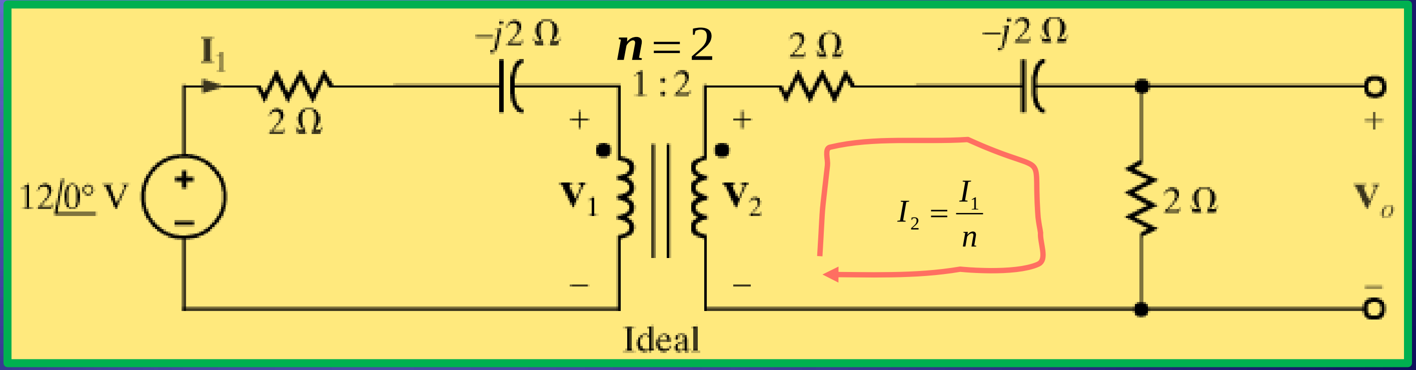
The previous equations could be rewritten as:

$$\boxed{\mathbf{V_1} = \frac{\mathbf{V_2}}{n}\ ;\ \mathbf{I_1} = n\mathbf{I_2};\ \text{and}\ \ Z_1 = \frac{Z_2}{n^2}}$$

When using these equations, **care must be taken** regarding the **polarities of voltages**, the **directions of currents**, and the **positions of dots**.

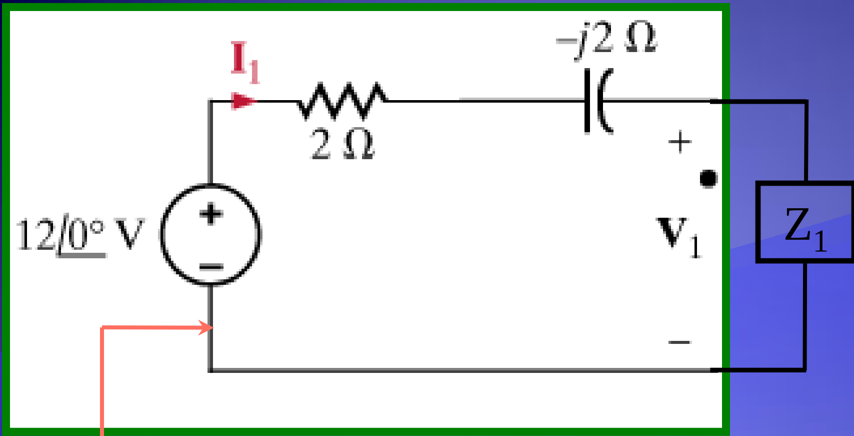
Example 10.8 – Voltages and Currents in an Ideal Transformer

E 10.6 – Reflecting Load Impedance into the Primary



Strategy: reflect impedance into the primary side and make transformer “transparent to user.”

$$Z_1 = \frac{Z_L}{n^2} = \frac{4 - j2}{4} = 1 - j0.5\Omega$$



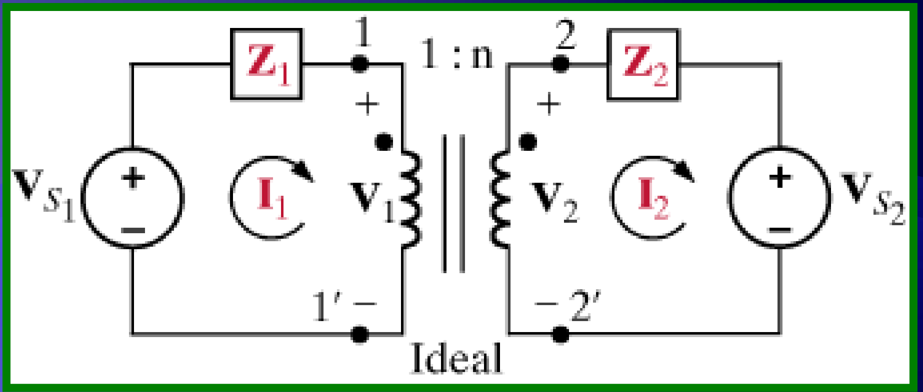
$Z_i = 3 - j2.5\Omega$

$$\mathbf{I}_1 = \frac{12\angle 0°}{3 - j2.5}$$

$$\mathbf{I}_1 = \frac{12\angle 0°}{3.91\angle -39.81°} = 3.07\angle 39.81°(\text{A})$$

Using Thevenin's Theorem: Reflecting into the Secondary

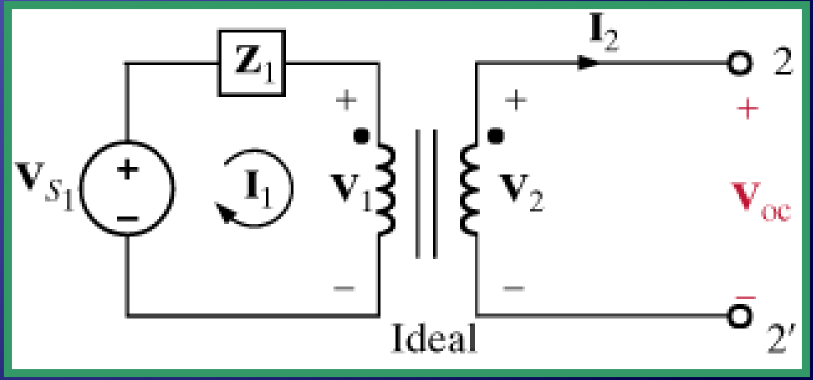
It is possible to use the **Thévenin or Norton Theorems** to replace the transformer and the primary circuit by their equivalent circuit.



Considering the voltage polarities and the current directions in the above circuit, we could write:

$$\mathbf{I}_1 = n\mathbf{I}_2;\ \text{and}\ \ \mathbf{V}_1 = \frac{\mathbf{V}_2}{n}$$

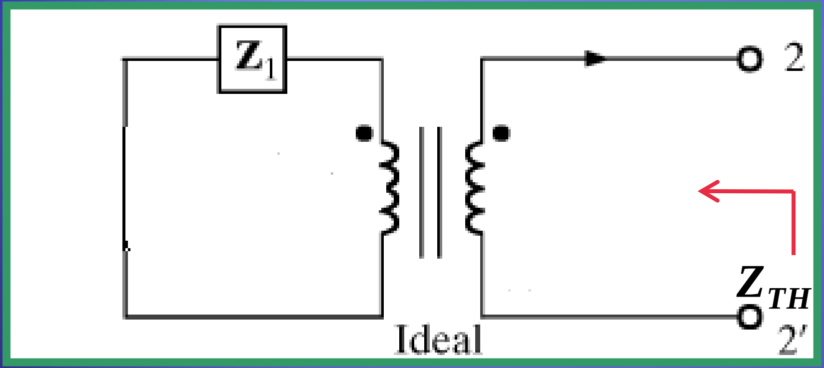
Looking into the circuit to the left of nodes 2-2′, we could write:



$$I_2 = 0 \Rightarrow I_1 = 0, \text{ and :}$$

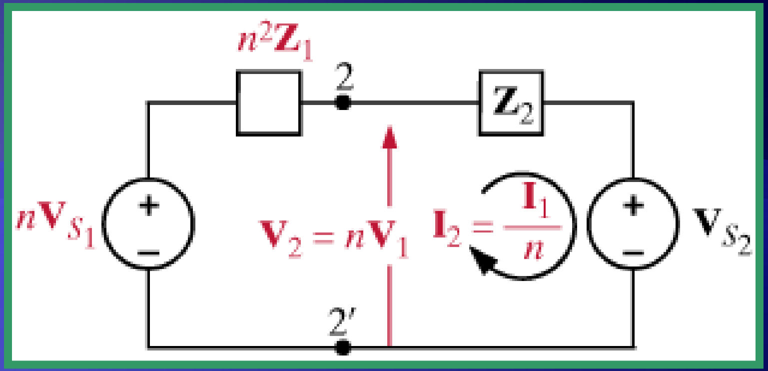
$$\mathbf{V}_{OC} = \mathbf{V}_2 = n\mathbf{V}_1 = n\mathbf{V}_{S_1}$$

On the other hand, the Thévenin equivalent impedance is calculated using the following circuit:



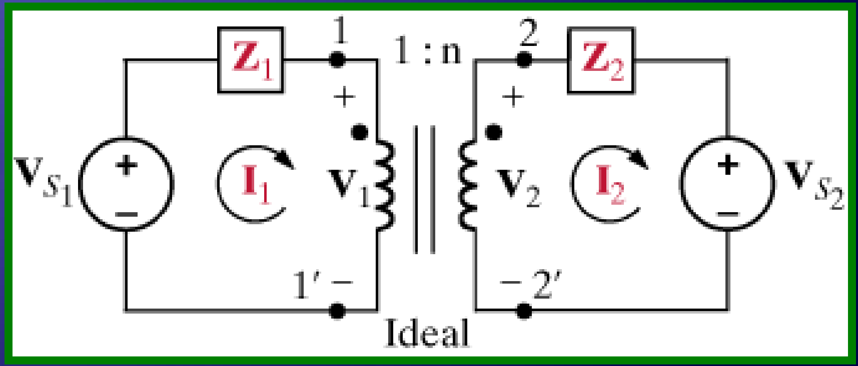
$$\Rightarrow Z_{Th} = n^2 Z_1$$

The resulting equivalent circuit with the **transformer made "transparent"** and **reflected**, along with the primary **into the secondary** becomes:



Using Thevenin's Theorem: Reflecting into the Primary

In a similar fashion, it is possible to use the **Thévenin Theorem** to replace the transformer and the secondary circuit by their equivalent circuit. In this case, we replace the circuit to the right of nodes 1-1′ :



In this case, we could write:

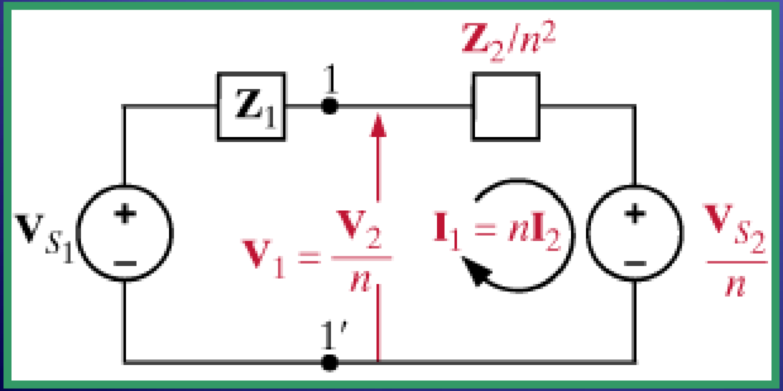
$$\boxed{\mathbf{I}_1 = 0 \Rightarrow \mathbf{I}_2 = 0, \text{and}:}$$

$$\boxed{\mathbf{V}_{\mathrm{OC}} = \mathbf{V}_1 = \frac{\mathbf{V}_2}{n} = \frac{\mathbf{V}_{S_2}}{n}}$$

On the other hand, the Thévenin equivalent impedance is the secondary impedance reflected into the primary:

$$\boxed{\Rightarrow Z_{\mathrm{Th}} = Z_1 = \frac{Z_2}{n^2}}$$

The resulting equivalent circuit becomes:



As a general rule, when finding the equivalent circuit for the transformer and its primary circuit, **each primary voltage is multiplied by *n*, each primary current is divided by *n*, and each primary impedance is multiplied by n^2**, and conversely.

However, **powers are the same** whether calculated on the primary or the secondary side.

We note that **if the location of one of the dots is different** from that shown in the present circuit, the turns ratio *n* will need to be replaced by –*n*.

We also note that **if the transformer circuit is different from the typical circuit** shown in the present circuit, Thévenin Theorem is typically applied to reduce the circuit to its present form.

Example 10.9 – Determining the Equivalent Circuits

PROBLEM SOLVING STRATEGY

Step 1: Examine the **voltage polarities and current directions** with respect to the transformer dots:

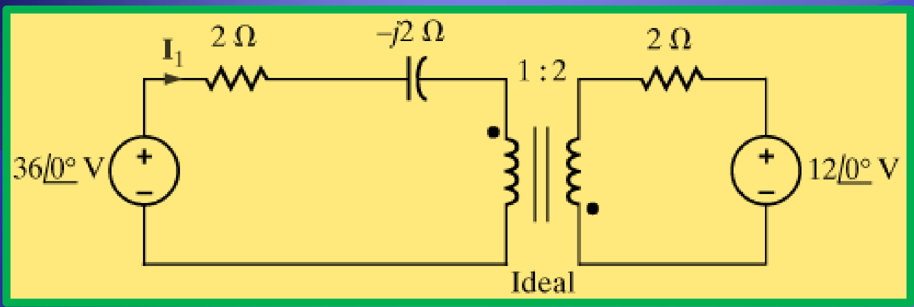
- If both voltages are **positive at the dots: $v_1/ v_2 = N_1/ N_2$,** otherwise: $v_1/ v_2 = -N_1/ N_2$
- If **one current enters the dotted terminal and the other leaves the dotted terminal**: **$i_1/i_2 = N_2/ N_1$,** otherwise: $i_1/i_2 = -N_2/ N_1$

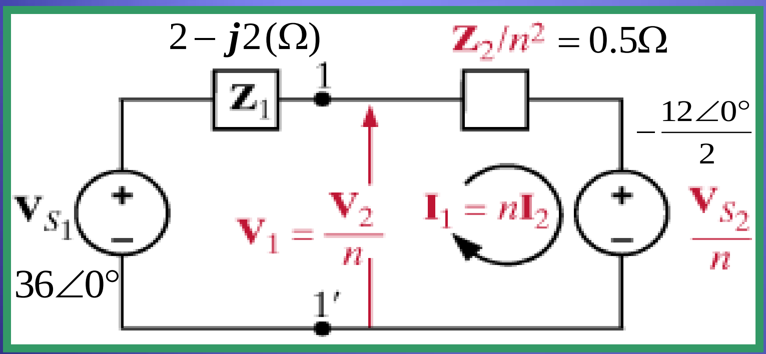
Step 2: For a typical circuit, made up of a single primary mesh and a single secondary mesh, **reflect the primary into the secondary** or vice versa, using the reflection equations, solve the resulting circuit using common circuit analysis techniques, then **reflect back to the original circuit** to determine the required variables – Note that impedances are scaled in magnitude only

Step 3: For more complex circuits, with unconnected windings, use Thévenin Theorem to reduce the circuit to a typical circuit.

Example 10.10 – Working With an Atypical Circuit

E 10.8 – Calculation of Primary Current in a Typical Circuit



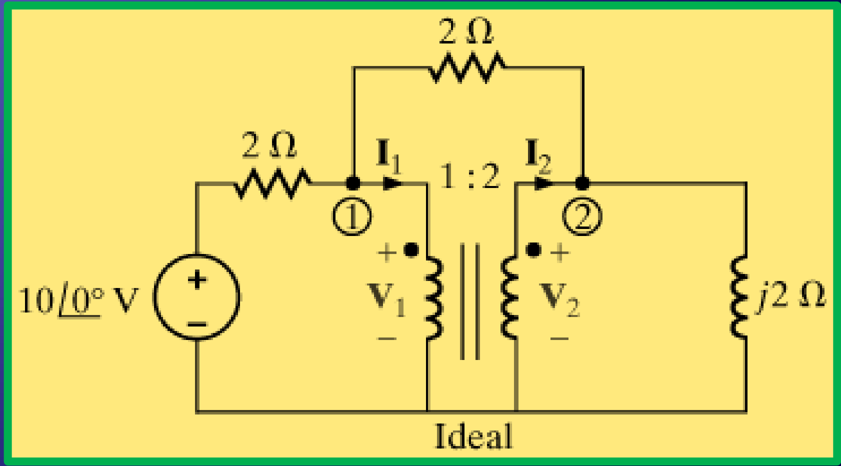


$$I_1 = \frac{36\angle 0° + 6\angle 0°}{2.5 - j2}$$

$$I_1 = \frac{42\angle 0°}{3.2\angle -38.66°}$$

We note that **if the ideal transformer has its windings electrically connected, the equivalent circuit techniques** (primary to secondary and vice-versa) **cannot, in general, be used**. Instead, nodal and mesh analysis is used, along with the transformer voltage and current transformation equations.

Example 10.11 – Ideal Transformer with Connected Windings



Phasor transformation equations for ideal Transformer:

$$V_1 = \frac{V_2}{2}$$
$$I_1 = 2I_2$$

Nodal equations:

$$@\ \text{Node}\,1: \ \frac{V_1 - 10\angle 0^\circ}{2} + \frac{V_1 - V_2}{2} + I_1 = 0$$

$$@\text{Node}\,2: \ \frac{V_2 - V_1}{2} + \frac{V_2}{j2} - I_2 = 0$$

Solving:

$$2\boldsymbol{V}_1 - \boldsymbol{V}_2 + 2\boldsymbol{I}_1 = 10\angle 0^\circ$$
$$-\boldsymbol{V}_1 + (1 - \boldsymbol{j})\boldsymbol{V}_2 - 2\boldsymbol{I}_2 = 0$$
$$\boldsymbol{V}_2 = 2\boldsymbol{V}_1$$
$$\boldsymbol{I}_1 = 2\boldsymbol{I}_2$$

$$\Rightarrow \boldsymbol{I}_1 = 5\angle 0^\circ$$

$$\boldsymbol{I}_2 = 2.5\angle 0^\circ$$

$$-\boldsymbol{V}_1 + (1 - \boldsymbol{j})(2\boldsymbol{V}_1) = 5\angle 0^\circ$$

$$\boldsymbol{V}_1 = \frac{5\angle 0^\circ}{1 - \boldsymbol{j}2} = \frac{5\angle 0^\circ}{2.24\angle -63.43^\circ}$$

$$\Rightarrow V_1 = \sqrt{5}\angle 63.43^\circ \text{ V}$$

$$\text{and } \ V_2 = 2\sqrt{5}\angle 63.43^\circ \text{ V}$$

END OF CHAPTER